Understanding the influence of real factors on leakage rates through cracks in 'leak-before-break' assessments

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LbB Concept

- Defense-in-depth argument used in the Nuclear Industry
- If there is a crack in a pressurised component, can a leak be detected before that crack reaches a critical size?
- Detection based on sensitivity of containment building detection equipment

Leakage rates

- Affected by fluid and structural issues;
- Critical Crack Size
- Crack Opening Area
- Calculate leak rate
- Is leak rate detectable?
    - If yes, LbB case can be made
    - If no, LbB case cannot be made
- More complicated with crack growth mechanisms
- Large safety factors (up to 10) put on leak rates at present - can these be reduced?

Aim

- Understand the influence of heat flux from leaking fluid to crack wall
- Thermal expansion affects crack opening area
- Feedback loop to find a steady state solution for the coupled thermo-mechanical problem
- Numerical simulation with the Extended Finite Element Method
- Provide a better understanding of leak rates through cracks to reduce safety factors in LbB assessments

Extended Finite Element Method

- Based on the partition of unity method [1]
- Standard shape functions enriched by additional terms
- Captures jumps and crack tip fields via step function and branch functions
- Does not require mesh to conform to geometry of discontinuity

$$U = \sum_{i \in I} N_i u_i + \sum_{j \in J} N_j H(x) a_j + \sum_{k \in K} N_k \psi(x) \quad (1)$$

- $H(x)$ is the Heaviside step function, a_j are the additional degrees of freedom associated with the node j, $\psi(x)$ are the branch functions at the crack tip.

Cohesive Crack

- Facilitates modelling in 1-d
- Based on methods proposed by Afsberg [2] and Unger [3]

$$t_c = k_c[[U]] \tag{2}$$

- where $[[U]]$ is the jump in displacement field across the crack, k_c is a modified stiffness parameter

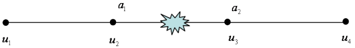

Figure: Three element structure with crack in second element. The displacement u_i are the standard finite element displacements, the a_i are the extra degrees of freedom associated with the cut element.

Cohesive Crack

- Heaviside step function captures jump in displacement

$$H(x) = \begin{cases} 1 & \text{when } x \in \Omega_+ \\ 0 & \text{when } x \in \Omega_- \end{cases} \tag{3}$$

- where Ω_- is on the left of the crack, and Ω_+ is on the right
- Discontinuity means that care must be taken with integration, simple to split up integration in 1-d analytical case, more complicated for 3-d numerical integration schemes.

Structure Stiffness Matrix

- The structure stiffness matrix for the arrangement shown in Fig. 1 is as follows:

$$K_{str} = \begin{pmatrix} \frac{EA}{L} & -\frac{EA}{L} & 0 & 0 & 0 & 0 \\ \frac{-EA}{L} & \frac{2EA}{L} & \frac{-EA}{L} & 0 & \frac{EA}{L} & \frac{-EA}{L} \\ 0 & \frac{-EA}{L} & \frac{2EA}{L} & \frac{-EA}{L} & \frac{EA}{L} & 0 \\ 0 & 0 & \frac{-EA}{L} & \frac{EA}{L} & 0 & \frac{EA}{L} \\ 0 & \frac{EA}{L} & \frac{-EA}{L} & 0 & \frac{EA}{L} + k_c/4 & \frac{-EA}{2L} + k_c/4 \\ 0 & \frac{-EA}{L} & 0 & \frac{EA}{L} & \frac{-EA}{2L} + k_c/4 & \frac{3EA}{2L} + k_c/4 \end{pmatrix}$$

- This is a standard linear elastic stiffness matrix for a 3 element 1-d system plus the additional cohesive terms in the right hand bottom corner.

Convection Through Crack

- Heat transfer from fluid to crack faces

$$\dot{q}'' = h(T_{wall} - T_{bulk}) \tag{4}$$

- Heat transfer correlation h obtained from correlations such as Dittus-Boelter

$$Nu = 0.023Re^{0.8}Pr^{n} \tag{5}$$

- Reynolds number requires mass flow rate of escaping fluid which can be obtained from equations in literature
- R6 provides a single phase equation for velocity

$$V = C_D \left(\frac{p_0}{\rho_0}\right)^{1/2} \tag{6}$$

- C_D is a discharge coefficient depending on friction and ciritcal flow conditions

Thermoelasticicity

- Thermal expansion on structure
- Thermal stresses and strains given by the following [4]:

  $$\epsilon_T = \alpha(T - T_0)I \tag{7}$$

  where T_0 is a reference temperature, α is the thermal expansion coefficient, I is the identity matrix. The corresponding stress is therefore:

  $$C : (\epsilon - \epsilon_T) = \sigma \tag{8}$$

- C is the elasticity matrix, which for the 1-d case can just be taken as E, the elastic modulus

Components of Stiffness matrix

$$\begin{pmatrix} K_{ij}^{uu} & K_{ij}^{ua} \\ K_{ij}^{au} & K_{ij}^{aa} \end{pmatrix} \begin{pmatrix} u_j \\ a_j \end{pmatrix} = \begin{pmatrix} f_i^u \\ f_i^a \end{pmatrix} \tag{9}$$

$$K_{ij}^{uu} = \int_\Omega (B_i^u)^T C B_j^u \, d\Omega \tag{10a}$$

$$K_{ij}^{ua} = \int_\Omega (B_i^u)^T C B_j^a \, d\Omega \tag{10b}$$

$$K_{ij}^{au} = \int_\Omega (B_i^a)^T C B_j^u \, d\Omega \tag{10c}$$

$$K_{ij}^{aa} = \int_\Omega (B_i^a)^T C B_j^a \, d\Omega + \int_{\Gamma_{coh}} (\bar{B}_i^a)^T k_c (\bar{B}_j^a) \tag{10d}$$

$\bar{B}_i^a$ are the difference in shape functions at the crack Γ_{coh}

$$f_i^u = \int_{\Gamma_t} (N_i^u) \bar{t} \tag{11a}$$

$$f_i^a = \int_{\Gamma_t} (N_i^a) \bar{t} \tag{11b}$$

Conductivity Matrix

- Thermal case is analogous to mechanical one
- Replace elastic modulus with thermal conductivity, and displacement with change in temperature
- Use Fourier's law

Coupled

- Coupling the mechanical and thermal problems gives the following expression:

$$f = K_{str}(u - u_T) \tag{12}$$

- The finite element equations to be solved are now:

$$K_{str}u = f + K_{str}u_T \tag{13}$$

- $u_T = \alpha LT$, and T is the solution of the thermal finite element equations.

MATLAB code

- Integration is performed analytically
- A MATLAB code solves the 1-d problem
- Boundary conditions are a force on fourth node for mechanical problem, and heat flux on end node for thermal problem
- Coupled model there is a 'constrained' and 'unconstrained' version. The constrained version limits the displacement on the end node so the expansion causes a compressive force, resulting in crack closure. Whereas the unconstrained version allows unlimited expansion.
- Get crack closure in both cases when there is a heat flux boundary condition at the crack

Table: Material Properties for Structure

Property	Value
Thermal Conductivity, k	$20W/K.m$
Cross Sectional Area, A	$1m^2$
Length of Element, L	$0.01m$
Young's modulus, E	$2GPa$
Cohesive Stiffness Parameter, K_c	$3GPam$
Initial temperature T_0	300^oC
Thickness, t,	$0.1m$

Table: Fluid Properties for Coolant

Property	Value
Crack Inlet Pressure, p_0	$16MPa$
Crack Outlet Pressure p_{ex}	$10MPa$
Density ρ	$650kg/m^3$
Viscosity, μ	$0.001Pa.s$
Length of crack, L_d	$0.01m$
Thermal Conductivity, k_f,	$0.01W/m.K$
Roughness Parameter, Ra,	$8 \times 10^{-6}m$
Specific Heat Capacity, C_p	$5kJ/kg$
Divergence Parameter, d	0.9
Average COD, W	$0.001m$

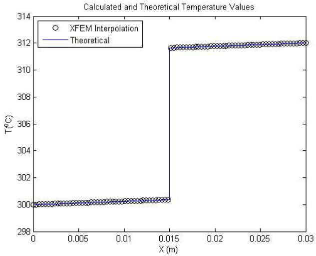

Figure: Temperature in cracked bar with fluid escaping

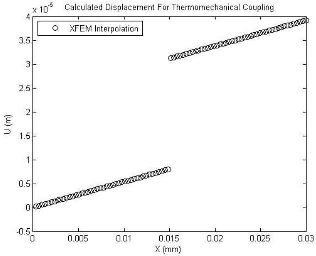

Figure: Unconstrained cracked bar under low heat flux.

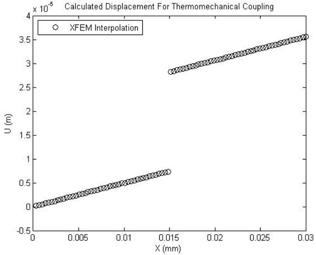

Figure: Constrained cracked bar under low heat flux, neglible crack closure.

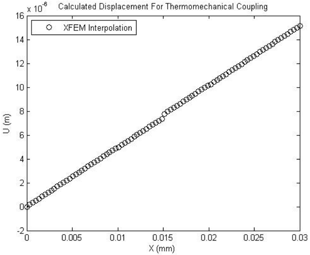

Figure: Constrained cracked bar under high heat flux, complete crack closure.

Conclusions

- 1-d mechanical problem can be modelled with cohesive crack, traction separation law
- Thermal problem models discontinuity with convective heat transfer across crack
- Coupled model uses thermoelasticity theory with additional thermal strain terms
- High heat fluxes give crack closure and reduced leak rate

Further Work

- Continue with 1-d imposing heat flux boundary conditions on crack faces
- Extend to 2-d case
- Incorporate more detailed fluid equations with pressure and temperature changes through thickness
- Iterate coupled problem to find steady state condition for crack - non linear
- 3-d case in Code-Aster, comparison with Ecrevisse subroutine for more complex geometries
- Recommendations for R6 code

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Any Questions?