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An Introduction to Nodal Methods for Reactor Physics

UNTF 2010

Sheldon Hall

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Introduction

The Reactor Core

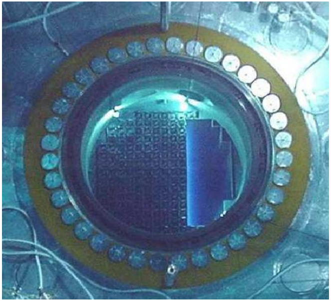

Figure: www.eia.doe.gov

- This is a pressurised water reactor (PWR) core.
- A third of the assemblies have been removed.
- The blue glow just off centre is the Cherenkov radiation.

We need to know:

- What happens when we take the control rods out?
- How do we best use the fuel?
- In an accident what might happen?

A Fuel Assembly

Figure: www.eia.doe.gov

Cannot practically model the full geometry:

- Complex assembly structure.
- Large core.
- Cannot measure conditions accurately enough.

The Neutronics Code

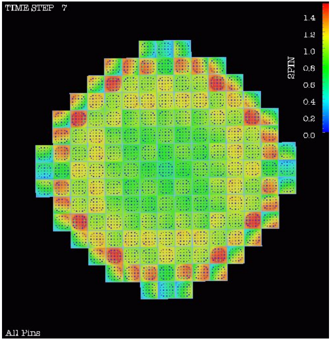

Figure: www.research.ibm.com

The aim of a neutronics code:

- To predict the spatial distribution of the power generated.

This facilitates:

- Determining the enrichment of the fuel and the amount of water surrounding it.
- Calculation of transients, both normal and accidental.
- Optimising the fuel management.
- Core-follow operations.
- Simulators.

The Neutron Transport Equation

The Angular Flux

The unknown which we are trying to determine is the angular flux

Definition

$\psi(\mathbf{r}, E, \mathbf{\Omega}, t) \equiv v\, N(\mathbf{r}, E, \mathbf{\Omega}, t) =$ the angular flux, where v is the neutron speed and N is the particle density distribution.

Neutron Transport Equation

The integro-differential form of the transport equation is given by:

$$\frac{1}{v}\frac{\partial\psi}{\partial t} = -\sigma_t(\mathbf{r}, E)\psi - \mathbf{\Omega}\cdot\nabla\psi + \int_{E'}\int_{\mathbf{\Omega}} \sigma_s(\mathbf{r}, E' \to E, \mathbf{\Omega}'\cdot\mathbf{\Omega})\psi(\mathbf{r}, E', \mathbf{\Omega}', t)\, dE'\, d\Omega' + \chi(E)\int_{E'} \nu\sigma_f(\mathbf{r}, E')\phi(\mathbf{r}, E', t)\, dE' + s$$

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Neutron Transport Equation

The integro-differential form of the transport equation is given by:

$$
\begin{aligned}
\frac{1}{v}\frac{\partial \psi}{\partial t} = & -\sigma_t(\mathbf{r}, E)\psi - \mathbf{\Omega}\cdot\nabla\psi \\
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\end{aligned}
$$

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$$+ \chi(E) \int_{E'} \nu\sigma_f(\mathbf{r}, E')\phi(\mathbf{r}, E', t)\, dE' + s$$

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Nodal Methods

Neutron Diffusion

$$\frac{1}{v}\frac{\partial \phi}{\partial t} = -\sigma_t \phi - \nabla \cdot \mathbf{J} + q$$
$$\mathbf{J} = -\frac{1}{3\sigma_t}\nabla \phi$$

- Assuming an isotropic distribution of neutron directions yields the neutron diffusion equation.
- The first equation is a neutron balance.
- The second is an approximation to the current.

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The Nodal Method

$$-\nabla \cdot \frac{1}{3\sigma_t}\nabla\phi(x,y,z) + \sigma_t\phi(x,y,z) = q$$

- Substituting one equation into the other we get a second order pde.
- This is further simplified by considering a steady state situation.

Transverse Integration

$$-\frac{\partial}{\partial x}\left[\frac{1}{3\sigma_t}\frac{\partial}{\partial x}\bar{\phi}(x)\right]+\sigma_t\bar{\phi}(x)=\bar{q}+L(x)$$

- The diffusion equation is integrated within a node, transverse to a direction under consideration.
- The leakage term is written as an unknown function of that direction.

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The Leakage Approximation

$$L(x) = L_0 + L_1 x + L_2 x^2$$

- An assumption is now made that the Leakage can be well represented by a quadratic function.
- This has no physical basis, it is used purely because it works.

Form of Transverse Integrated Equation

$$-\frac{\partial}{\partial x}\left[\frac{1}{3\sigma_t}\frac{\partial}{\partial x}\bar{\phi}(x)\right] + \sigma_t\bar{\phi}(x) = \bar{q} + L_0 + L_1 x + L_2 x^2$$

There are now a few approaches that can be taken to construct the numerical method:

- Assume that the 1-D flux can be approximated by a polynomial.
- Solve the 1-D equation exactly within a node.
- Use a mixture of the other two.

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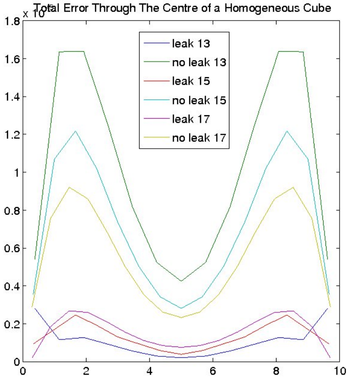
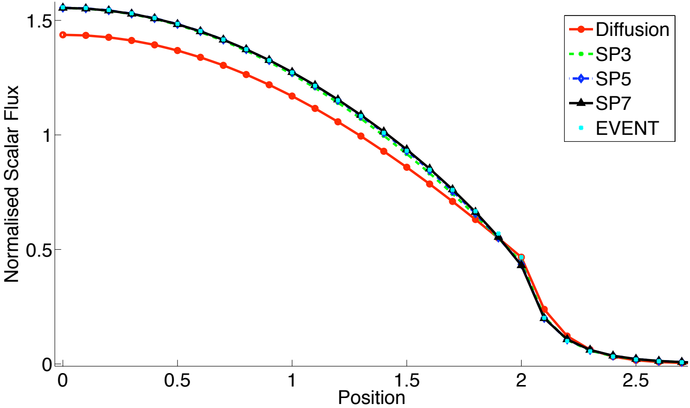
Results



Supercell

	A	A	A	A	A	A	A	A	A	B	B	B	B	B	B	B	B	B	
1	DET	U2	U2	WH	U2	U2	WH	U2	U2	M4.3	M7.0	WH	M7.0	M8.7	WH	M8.7	M8.7	DET	1
1	U2	U2	U2	U2	U2	U2	U2	U2	U2	M4.3	M7.0	M7.0	M7.0	M8.7	M8.7	M8.7	M8.7	M8.7	1
1	U2	U2	U2	U2	U2	U2	U2	U2	U2	M4.3	M7.0	M7.0	M7.0	M8.7	M8.7	M8.7	M8.7	M8.7	1
1	WH	U2	U2	WH	U2	U2	WH	U2	U2	M4.3	M7.0	WH	M7.0	M8.7	WH	M8.7	M8.7	WH	1
1	U2	U2	U2	U2	U2	U2	U2	U2	U2	M4.3	M7.0	M7.0	M7.0	M7.0	M8.7	M8.7	M8.7	M8.7	1
1	U2	U2	U2	U2	U2	WH	U2	U2	U2	M4.3	M7.0	M7.0	WH	M7.0	M7.0	M7.0	M7.0	M7.0	1
1	WH	U2	U2	WH	U2	U2	U2	U2	U2	M4.3	M7.0	M7.0	M7.0	M7.0	WH	M7.0	M7.0	WH	1
1	U2	U2	U2	U2	U2	U2	U2	U2	U2	M4.3	M4.3	M7.0	M7.0	M7.0	M7.0	M7.0	M7.0	M7.0	1
1	U2	U2	U2	U2	U2	U2	U2	U2	U2	M4.3	M4.3	M4.3	M4.3	M4.3	M4.3	M4.3	M4.3	M4.3	1
2	M4.3	M4.3	M4.3	M4.3	M4.3	M4.3	M4.3	M4.3	M4.3	U2	U2	U2	U2	U2	U2	U2	U2	U2	2
2	M7.0	M7.0	M7.0	M7.0	M7.0	M7.0	M7.0	M4.3	M4.3	U2	U2	U2	U2	U2	U2	U2	U2	U2	2
2	WH	M7.0	M7.0	WH	M7.0	M7.0	M7.0	M7.0	M4.3	U2	U2	U2	U2	U2	WH	U2	U2	WH	2
2	M7.0	M7.0	M7.0	M7.0	M7.0	WH	M7.0	M7.0	M4.3	U2	U2	U2	WH	U2	U2	U2	U2	U2	2
2	M8.7	M8.7	M8.7	M8.7	M7.0	M7.0	M7.0	M7.0	M4.3	U2	U2	U2	U2	U2	U2	U2	U2	U2	2
2	WH	M8.7	M8.7	WH	M8.7	M7.0	WH	M7.0	M4.3	U2	U2	WH	U2	U2	WH	U2	U2	WH	2
2	M8.7	M8.7	M8.7	M8.7	M8.7	M7.0	M7.0	M7.0	M4.3	U2	U2	U2	U2	U2	U2	U2	U2	U2	2
2	M8.7	M8.7	M8.7	M8.7	M8.7	M7.0	M7.0	M7.0	M4.3	U2	U2	U2	U2	U2	U2	U2	U2	U2	2
2	DET	M8.7	M8.7	WH	M8.7	M7.0	WH	M7.0	M4.3	U2	U2	WH	U2	U2	WH	U2	U2	DET	2
	A	A	A	A	A	A	A	A	A	B	B	B	B	B	B	B	B	B	

Solutions of SPN



Conclusions

- Simplify the neutron transport equation into a diffusion equation.
- Instead of directly discretising 3D equations, transverse integrate to obtain 3 1D equations.
- Approximate the coupling term with a quadratic.
- Solve each 1D equation analytically.
- For a better solution retain more terms from the transport equation.

Acknowledgements

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British Energy
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Are there any questions?